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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Improved First-Order Approximation of Eigenvalues and Eigenvectors"

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THE authors of Ref. 1 present a method for using a reduced basis to better approximate eigenvalues and eigenvectors that Murthy and Haftka have previously published as the reduction method approximation.[2] Equations (9–11) in Ref. 1 are equivalent to Eqs. (8–10) in Ref. 2. They differ in notation and in the use of real, symmetric matrices for self-adjoint structural problems in Ref. 1, compared with the use of complex, general matrices for non-self-adjoint problems in Ref. 2. Hence, the method of Ref. 1 is a special case of the reduction method approximation. In contrast to the title of Ref. 1, the authors of Ref. 2 demonstrate that the reduction method is actually a third-order method.

The two references differ in one further aspect, namely, in selecting the root of the resulting 2×2 reduced-basis, linear eigenvalue problem. In Ref. 1 the authors use five heuristic criteria, whereas Murthy and Haftka choose the root closest to the linear (first-order) Taylor-series approximation of the eigenvalue (comparable to the second of the five criteria in Ref. 1).

An observation may improve slightly the heuristic algorithm for determining the eigenvector approximation. The reduction-method approximation contains all of the ingredients to calculate the second-order Taylor-series approximation (TSA2) of the eigenvalue. The TSA2 of the eigenvalue

$$\lambda_i \cong \lambda_i^0 + \sum_{j=1}^{p}\left(\frac{\partial \lambda_i}{\partial x_j}\right)\Delta x_j + \frac{1}{2}\sum_{j=1}^{p}\sum_{k=1}^{p}\left(\frac{\partial^2 \lambda_i}{\partial x_j \partial x_k}\right)\Delta x_j \Delta x_k \tag{1}$$

requires second derivatives of the eigenvalue with respect to design variables. However, the second derivative, in turn, requires only first-order derivatives of the eigenvalue, mass, stiffness, and associated eigenvector, all of which are available in Eqs. (6–8) of Ref. 1. Following the notation of Ref. 1, the eigenvalue second derivative used in Eq. (1) may be expressed as[3]

$$\frac{\partial^2 \lambda_i}{\partial x_j \partial x_k} = \left[\{\phi^0\}_i^T([F_{i,j}]\{\phi_{i,k}\} + [F_{i,k}]\{\phi_{i,j}\}) - \left(\frac{\partial \lambda_i}{\partial x_j}\{\phi^0\}_i^T\frac{\partial [M^0]}{\partial x_k} + \frac{\partial \lambda_i}{\partial x_k}\{\phi^0\}_i^T\frac{\partial [M^0]}{\partial x_j}\right)\{\phi^0\}_i\right]\Bigg/ \{\phi^0\}_i^T[M^0]\{\phi^0\}_i \tag{2}$$

where

$$[F_{i,j}] = \frac{\partial [K^0]}{\partial x_j} - \lambda_i\frac{\partial [M^0]}{\partial x_j} - \frac{\partial \lambda_i}{\partial x_j}[M^0] \tag{3}$$

Knowing the TSA2 of the eigenvalue can simplify the selection of the appropriate root for the reduced-basis scaling parameters in Eq. (9) of Ref. 1. The root closest to the TSA2 of the eigenvalue should be chosen for use in Eq. (19) of Ref. 1. This suggests an interpretation for the reduced-basis eigenproblem. Substituting the current Eq. (1) into Eq. (11) of Ref. 1 does not, in general, allow a nontrivial solution of the 2×2 reduced-basis equations for the scaling parameters of the perturbed eigenvector. Choosing the root of Eq. (4) of Ref. 1 closest to λ_i in Eq. (1) implies an associated eigenvector approximation in the space spanned by the unperturbed eigenvector and its first-order differential, i.e., Eq. (5) of Ref. 1. Thus, the reduced basis, consisting of the zero- and first-order eigenvectors, approximates the TSA2 of the eigenvector. The corresponding root represents a third-order approximation of the eigenvalue.

References

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[2]Murthy, D. V., and Haftka, R. T., "Approximations to Eigenvalues of Modified General Matrices," *Computers and Structures*, Vol. 29, No. 5, 1988, pp. 903–917.

[3]Miura, H., and Schmit, L. A., Jr., "Second Order Approximation of Natural Frequency Constraints in Structural Synthesis," *International Journal for Numerical Methods in Engineering*, Vol. 13, No. 2, 1978, pp. 336–351.

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Reply by the Authors to R. A. Canfield

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THE authors appreciate the comments made by Canfield and his bringing to our notice the equivalence between the method presented in Ref. 1 with an earlier study published in Ref. 2.

Before addressing the issues raised by Canfield, the authors would like to reemphasize some of the motivations behind Ref. 1. The main objective of the work was to present a general method for approximating the eigenvalues and eigenvectors of modified structural dynamical systems. In contrast to most earlier studies, an important focus was to develop a method that could be applied to approximate the low-medium-frequency eigenmodes for moderate to large magnitudes of perturbations in the system matrices, which we needed

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